SLIP with swing leg augmentation as a model for running

Aida Mohammadi Nejad Rashty, Maziar Ahmad Sharbafi and Andre Seyfarth

***Abstract*— Swing leg adjustment, repulsive leg function and balance are key elements in the control of bipedal locomotion. In simple gait models like spring-loaded inverted pendulum (SLIP), swing leg control can be applied to achieve stable running. The aim of this study is to investigate the ability of pendulum like swing leg motion for stabilizing running and reproducing a desired (human like) gait pattern. The employed running model consists of two sub-models: SLIP model for the stance phase and a pendulum based control for the swing phase. It is shown that with changing the pendulum length at each step, stable running gaits with widely different performances are achieved. The body vertical speed at take off is utilized as feedback information to tune the pendulum length as the control parameter. In particular, the effect of the pendulum length adjustment on the motion characteristics like horizontal speed, apex height and the stabilized system energy will be investigated. With this method key features of the human like swing leg motion e.g. leg retraction can be reproduced. Higher speeds correspond larger angular motion of each leg which is in agreement with experimental results in previous studies. The presented model also explains the swing-leg to stance-leg interaction mechanism which was not addressed in the underlying SLIP model. This conceptual model can be considered as a functional mechanical template for legged locomotion and can be used to build more complex models, e.g. having segmented legs or an upper body.**

I. INTRODUCTION

Stabilization of the locomotion as the main issue for the control of bipedal robots can be divided to leg swinging, bouncing and balancing. Due to the complexity of the robots and, of course humans, the implementation of stabilizing strategies is a challenge. However, fundamental strategies to gain stability can be deduced from very simple simulation models. Rebounding on compliant legs is one of the basic mechanical consideration in human locomotion, especially running [1]. Simple conceptual models, called "templates" [2] have proved to be very helpful for describing and analyzing of animal/human locomotion. On the other hand, several bipedal robots were developed based on conceptual modeling of human locomotion to produce a robust and efficient movement [3], [4], [5].

As one of the fundamental template models, the spring-loaded inverted pendulum (SLIP) model [6][7] consisting of a point mass atop a massless spring describes basic features of human gaits (walking [1], hopping and running [6][8]) very well. Another interesting property of the SLIP is its asymptotic stability against perturbations conserving energy, even with a constant angle of attack [8]. In this model with passive mechanism during contact times, the motion stability only depends on the leg adjustment approach. Among studies investigating the influence of kinematic conditions on running motion stability like [9], a few models suggest strategies for stablizing running or hopping movements [10], [11], [12], [13], [14]. In [15], different leg adjustment strategies that provide stability over a broad range of running patterns are considered and compared to human running data. It is shown that adaptation of leg parameters (length and angle) matches to a specific leg stiffness adjustment. Therefore, in addition to the necessity of having two different controllers to adjust leg parameters, synchronizing them makes it more complex.

In [16], the influence of a pendulum-like swing leg movement on the stability of spring-mass forward hopping was investigated. This method inherited the idea of having an oscillatory swing leg motion by addition of a torsional spring to the hip joint in passive walker [12]. Such a passive structure also results in an elastically enforced pendulum motion during swing phase. In the Knuesel et al. paper [16], the model just had one leg and the pendulum motion was utilized as the leg adjustment and the swing leg motion simulation during flight phase was not the goal. The result of these assumptions is forward hopping (not running) with high apex height which is not practically feasible.

A more comprehensive application of pendulum like motion for swinging the leg (two pendulum in flight and one in stance) is presented for running in this paper. Combination of SLIP model and pendulum motion gives the possibility to define both legs configuration during whole motion. The focus of this paper is presenting a conceptual model to produce human like swing leg movement beside the stability of motion which is not addressed in previous researches. Additionally, we identified several types of running patterns, e.g. symmetric and asymmetric running, that accounts for high variability of gait. In that respect, the only control parameter is pendulum length adjustment which is done by an event based control using vertical body speed at takeoff moment. In summary, a combination of SLIP and pendulum resembles the swing leg motion and provides the stability.

II. METHODS

A. Running model

The model splits the motion to two continuous phases (*flight* and *stance*) which switch to each other by a discrete mapping. With this model, running and hopping could be explained by the equations defined in the following subsections. Note that this model is a conceptual model to describe human locomotion (like SLIP), not a physical model.

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A. Mohammadi Nejad, M. A. Sharbafi and A. Seyfarth are with Lauflabor Laboratory, Technical University of Darmstadt, Darmstadt, Germany, {aidamnr, sharbafi, seyfarth}@sport.tu-darmstadt.de

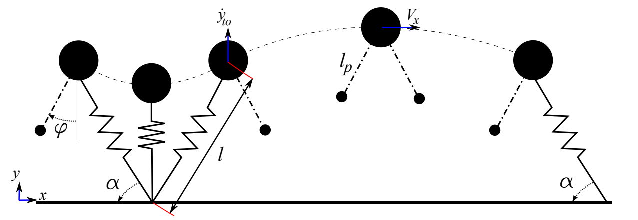

Fig. 1. Stance and flight phases of running modeling using SLIP and pendulum motion. The pendulum motion just determines the leg direction and has no effect on CoM motion. Dashed lines are applied to show virtual pendulums which have no interaction with CoM movement.

1) Flight phase: In flight phase, the legs do not touch the ground and the Center of Mass (CoM) moves in a ballistic motion. Since the legs are massless, their motions never affect the CoM movement. However, both legs motions are represented by two separate single pendulums with static pivot points (see Fig.1). The equations of single pendulum motion which are used for the swinging legs, are as follows:

$$\ddot{\varphi} = -\frac{g \sin \varphi}{l_p} \tag{1}$$

in which l_p and φ are the pendulum length (equal for both legs) and angle with respect to vertical axis as shown in Fig.1, respectively and g is the gravitational acceleration. Since the pendulum mass does not play any role in pendulum movement, considering massless legs (converging the pendulum mass to zero) and no interaction between the leg and body motion is valid[1].

With these two pendulums we want to resemble the front and hind legs motion during flight phase. The front leg pendulum, like e.g. a fixed angle of attack represents a possible leg adjustment strategy which is required for simulating locomotion with a SLIP model. After touch down occurrence, the front leg converts to the stance leg and the other leg remains a pendulum. The only parameter which influences the leg motion in flight phase is the pendulum length.

The model acts in the sagittal plane with x and y being the position of the center of mass. The CoM motion is defined by

$$\begin{aligned} \ddot{x} &= 0 \\ \ddot{y} &= -g \end{aligned} \tag{2}$$

2) Stance phase: In the stance phase, we have two legs, one of which is in contact with the ground and called stance leg. The stance leg is modeled by SLIP (Spring Loaded Inverted Pendulum) which is a massless spring with a mass on top of it. This point mass represents the body mass and can be considered as the CoM (Center of Mass) of the whole body[2]. The parameters of the spring-mass model were set similar to those used in [8]. As described in the previous section, the pendulum mass is not relevant in defining the motion (See Eq. (1)). Then the CoM motion is described by

$$\begin{aligned} \ddot{x} &= \tfrac{kx(l_0 - l)}{l} \\ \ddot{y} &= \tfrac{ky(l_0 - l)}{l} - mg \end{aligned} \tag{3}$$

in which x, y, l k and l_0 are the horizontal and vertical positions of the CoM, the leg length, the spring stiffness and rest length, respectively. The leg length is computed by $l = \sqrt{x^2 + y^2}$. Stance phase starts with touchdown (TD), the moment that the distal end of the leg hits the ground and ends with takeoff (TO) when the ground reaction force (GRF) has no vertical component.

3) Switching between stance and flight: A mapping between stance and flight models is required two times in each step; once in TO and once at TD. In take off moment, the front leg motion is continuation of the pendulum in stance leg. The initial conditions of the second pendulum which are determined by its angle (φ) and angular velocity ($\dot{\varphi}$) are adopted from the stance leg with the following equations:

$$\begin{aligned} \varphi &= \arctan \tfrac{x}{y} \\ \dot{\varphi} &= \tfrac{\dot{x}y - \dot{y}x}{l^2} \end{aligned} \tag{4}$$

This pendulum movement which replicates the hind leg motion, keeps the angle and angular velocity of the previously stance leg, as the second swinging leg in flight phase. Then, during flight phase, two pendulums define the legs' motions and the CoM motion is determined by the ballistic motion as described before.

At touch down moment which is detected when $l_0 \cos \varphi = y$, the front leg will change to the springy leg and the hind leg continues its pendulum motion. At this moment the stance leg initial conditions are defined by inheriting the CoM motion from flight phase dynamics (Like original SLIP model). Therefore, to switch from flight to stance, we just need to find the TD moment by the front leg orientation and vertical position of the CoM.

B. Event based control for pendulum length adjustment

In pendulum motion, the only parameter that can change the motion characteristics like frequency, is the pendulum length. With a constant pendulum length the model produces periodic motion that is not robust, i.e. stable limit cycles have small regions of attraction. Consequently, the resulting periodic motion is sensitive to parameters variations or perturbations (low robustness). To make the controlled system more stable and even robust, we present an event based pendulum length adjustment technique at each takeoff moment with the following equation:

$$l_p = l_{p0} \sqrt{\frac{\dot{y}_{to}}{\dot{y}_0}} \tag{5}$$

in which l_{p0} is the initial pendulum length and $\dot{y}_0$ and $\dot{y}_{to}$ are the absolute vertical CoM velocities at touch down and at take off, respectively.

[1]It is remarkable that this is a conceptual model (like SLIP) to describe the running motion characteristics and is not physically implementable. Therefore, in this model two completely separate motions are considered for leg and body movements.

[2]In human body, Sacrum which is less than 10 centimeter above hip point is considered as an acceptable approximation of CoM [17]. In SLIP model, a virtual leg between CoM and foot is approximated by spring.

In the following, the initial values are set to desired pendulum length and vertical speed which result in a limit cycle without adapting at each step. Hence, if the states of the system start from somewhere on the limit cycle, then $\dot{y}_{to}$ is equal to $\dot{y}_0$, the pendulum length remains equal to l_{p0} and the states stay on the limit cycle producing the periodic motion. On the other hand, if they leave the limit cycle, changing the pendulum length with respect to the ratio between the vertical velocity and its desired value can return it back to another limit cycle. This method of event based control results in a stable solution with a takeoff vertical speed in the neighborhood of the initial value $\dot{y}_0$, if $\dot{y}_1 < \sqrt{3}\dot{y}_0$ which means the vertical velocity at the first takeoff is close enough to $\dot{y}_0$. (See Appendix. A for the proof). It shows that the system is ultimately bounded in a sufficiently large neighborhood of the stable solution which supports deviation with magnitude 70% of the initial value.

C. System analysis

The simulations starts with touch down and a complete step is defined from TD to TD including one stance and one flight phase. Two sequential steps produce a stride, defined from touch down of one leg to the next touch down of the same leg. The stability is analyzed considering the n-step periodic motion which means having the same CoM states after n steps. Since with the proposed modeling and control approach, in many cases we obtain ultimate boundedness (especially for $n > 2$) instead of asymptotic stability, K-step stability is utilized [18], [16]. The controlled system is K-step stable (ultimately bounded) if it does not fall and never leaves a certain neighborhood of the periodic response in K steps. With this definition, a broader and maybe more applicable definition for mechanical stability is presented. On the other hand, with evaluating the eigenvalues of the Poincaré map, it is not possible to find the responses which are ultimately bounded but not asymptotically stable.

In order to detect the n-step periodicity of the motion, the algorithm which is shown in Fig. 2 is utilized. In this algorithm, the event is reaching the maximum height (apex), in which the vertical speed is equal to zero and the energy of the system can be computed by the horizontal speed and the hopping height. So, $V_x(i)$ and $y_a(i)$ are the horizontal velocity and CoM height at i^{th} apex. Through this algorithm, the motion is n-step periodic if the following conditions are held.

$$\begin{cases} |V_x(i) - V_x(i-n)| < \epsilon_v & \forall i,\ K-m \leq i \leq K \\ |y_a(i) - y_a(i-n)| < \epsilon_y & \forall i,\ K-m \leq i \leq K \end{cases} \quad (6)$$

Hence, considering constant numbers ϵ_v and ϵ_y close to zero, with this definition the system is n-step periodic if the horizontal speeds and hopping heights are repeated after every n steps. This definition is more precise than having the same energy in every n steps, when different combinations of speed (V_x) and height (y_a) can result in same energy level. The parameters of the model and periodicity detection are depicted in Table.I. Another relation which is checked in this paper is dependency of the horizontal speeds to the

```
For n = 1 : N
    isPeriodic = true;
    For i = K − m + 1 : k
        ΔV_x = V_x(i) − V_x(i − n);
        Δy_a = y_a(i) − y_a(i − n);
        If |ΔV_x| > ε_v or |Δy_a| > ε_y
            isPeriodic = false;
            break;
        end
    end
    If isPeriodic
        V̄_x = (1/n) Σ_{j=1}^{n} V_x(K − j + 1);
        ȳ_a = (1/n) Σ_{j=1}^{n} y_a(K − j + 1);
        E = (1/2) M V̄_x^2 + M g ȳ_a
        break;
    end
end
```

Fig. 2. The algorithm to detect the periodicity. If flag *isPeriodic* is true, the system is n-step periodic, otherwise we consider it as non-periodic. $\bar{V}_x$ and $\bar{y}_a$ are the average velocity and apex height of the last n steps. E is the energy of the system which is measured at apex.

TABLE I
MODEL PARAMETERS

Parameter	symbol	value [units]
Body mass	M	80 [kg]
Gravitational acceleration	g	9.81 [m/s^2]
pendulum length	l_p	0.1-0.25 [m]
leg stiffness	k	20000 [N/m]
leg rest length	l_0	1 [m]
Nominal apex height	y_a	1-1.25 [m]
Number of steps	K	100
Maximum number of steps for priodicity	N	10

pendulum length. To investigate the influence of changing pendulum length on motion speed, the correlation of these two variables are computed by:

$$corr V_x, l_p = \frac{cov(V_x, l_p)}{\sigma_{V_x}\sigma_{l_p}} \quad (7)$$

in which $cov(.,.)$ gives the covariance of two data variables and σ stands for the variance. Since, for 1-step periodic motions, there is no variation in speed and pendulum length in sequential apexes, this evaluation is performed for periodic motions with $n > 1$. Thus, increasing the pendulum length yields in slower motion[3] and consequently, steeper leg (smaller angle of attack) at touch down. It happens because of swing leg retraction which is observed already in human locomotion [19], [20], [21] and simulated models [22], [23], [13]. Eventually, smaller angle of attack reduces the forward speed [3] which means that the correlation between l_p and V_x should be around -1.

For comparison with human running, the leg angle during the complete stride is considered for different speeds.

[3]It is concluded from Eq.(1) and also with approximation of the pendulum motion by $\ddot{\varphi} = \frac{g}{l}\varphi$, the frequency of the motion is estimated as $\sqrt{\frac{g}{l}}$. It confirms that the pendulum length increment slows down the motion.

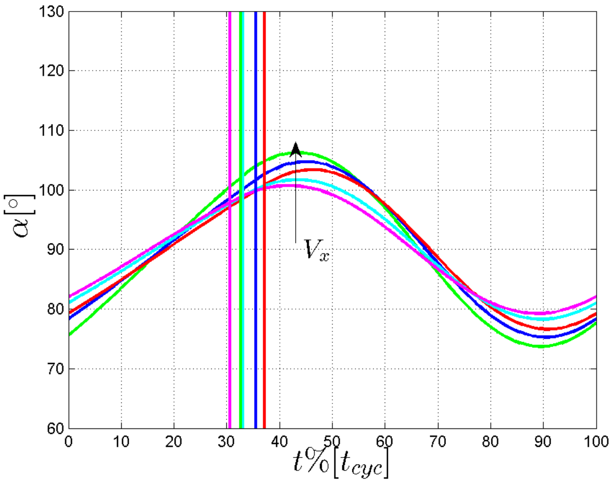

Fig. 3. Leg angle during a complete stride for different running speeds. Vertical lines show the takeoff occurrences which are between 31 to 37 percent of the gait cycle.

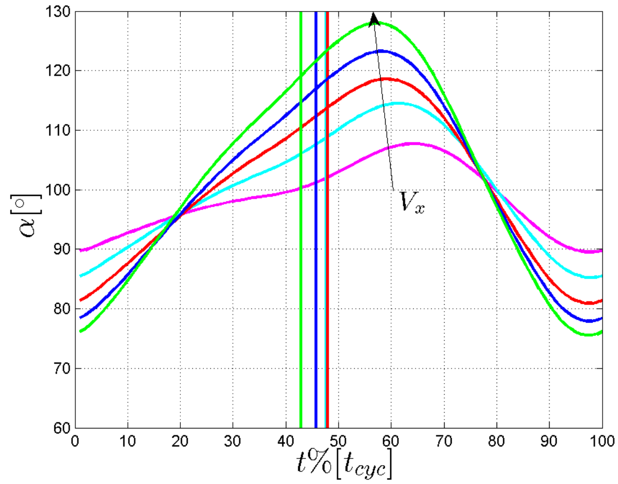

Fig. 4. The human leg angles during complete stride for different speeds. The plots are the average values for 21 subjects based on data, collected by Lepfert [24]. Vertical lines show the takeoff occurrences which are between 43 to 48 percent of the gait cycle.

III. RESULTS

Running characteristics and its relation to the proposed swing leg control parameters are investigated in this section. With model parameters stated in Table. I, different combinations of the initial conditions and pendulum lengths resulted in various types of running. First of all, for each horizontal speeds (V_x), the initial conditions are determined by the initial angle of attack (α_0), pendulum angle (φ_0)[4] and vertical speed ($\dot{y}_{to}$). With these assumptions, the dimension of search space is 5; three from initial conditions, one for running speed and one for initial pendulum length. Each combination of these parameters is simulated for 100 steps and the stability analysis is performed as described in the previous section.

A. Leg angle in running

Running with each speed could be produced by different combinations of the initial conditions and control parameters e.g. pendulum initial length and initial angle. In Fig. 3, the leg angle is drawn during the gait cycle for a complete stride. After touch down, the leg becomes more vertical and the leg angle increases until take off. This angle increment continues in flight phase. Shortly after takeoff, the leg stops going backward (retraction) and forward motion (protraction) starts. From this moment, the leg angle decreases until the last 10 percent of the gait cycle. Before the next touch down, leg retraction happens. This kind of swing leg retraction is a key feature in human motion [13]. Human leg behavior is also shown in Fig. 4 for different speeds. The human leg motions are qualitatively similar to the simulated model. Two peaks are observed in both figures and the range for the angles are also close to each other. In the proposed model, similar to human movement, the variations of the leg angle increase when the running speed increases. Ratio between stance and flight phases' durations is also similar in simulation and human experiments and changes with running speed.

B. Different running patterns

With the proposed modeling and control approach, several types of running patterns are identified with different periodicity (n extracted from the algorithm proposed in Fig. 2). Different trends of CoM motion in sagittal plane are shown in Fig. 5. This wide range of symmetric and asymmetric running patterns demonstrates the ability to produce high variability of the gaits.

In the first plot (Fig. 5,top-left) a sample of 1-step periodic

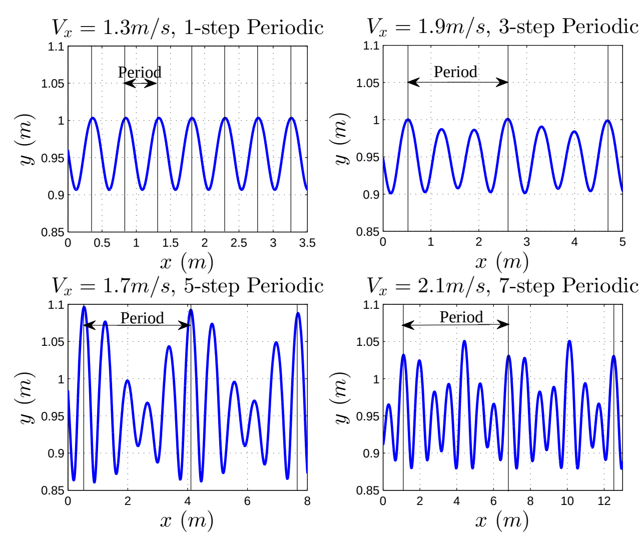

Fig. 5. Different hopping patterns of CoM movements. The periodicity changes from gait to gait. The time window illustrates at least two periods.

[4]The angular speed of the pendulum is set to zero because any combination of angle and negative angular velocity (which is needed for the swing leg) is achievable by changing the angle with zero angular velocity.

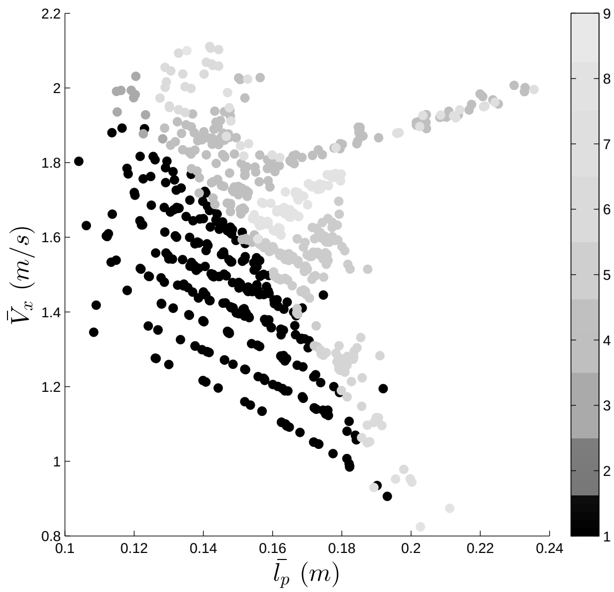

Fig. 6. Influence of pendulum length l_p on motion speed V_x. Darker points mean smaller number of gait periodicity.

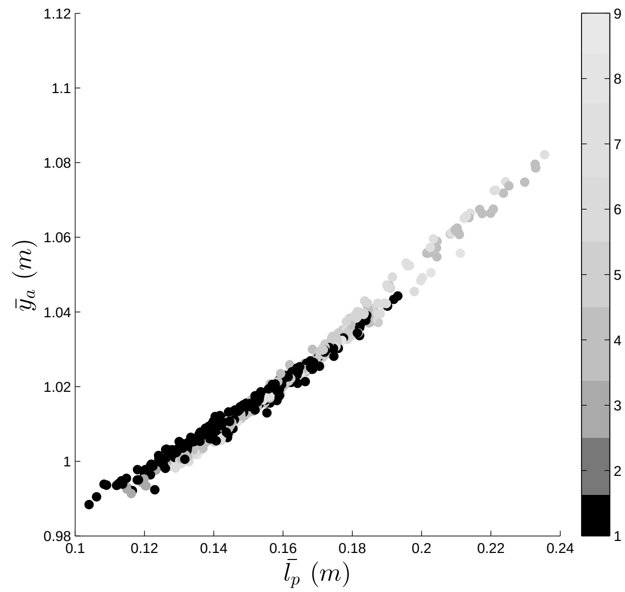

Fig. 7. Influence of pendulum length l_p on apex height y_a. Darker points mean smaller number of gait periodicity.

motion is depicted. In this motion, the CoM has the same vertical position after each step and the horizontal displacement is fixed at each step. In the next plot for 3-step periodicity (top-right), the apex height in the first step is higher than for the two next steps. This happens by having one short and two sequential larger steps. Different step lengths and apex heights can be observed in the other running patterns as shown in Fig. 5.

C. Pendulum length change effect

In this section, the influence of changing the pendulum length ($\bar{l}_p$) on motion speed ($\bar{V}_x$), apex height ($\bar{y}_a$) and system energy (E) is investigated (See Fig. 2 for the definitions). In Fig. 6, the points show the stable solutions for different average velocities and pendulum lengths. It is observed that for 1-step periodic motions, running between 1.2 m/s to 1.6 m/s can be performed with a large range of pendulum lengths. In general, reducing the pendulum length results in increasing the motion speed, as expected. Asymmetric running is achievable mostly by either increasing the pendulum length for slow motions or shortening it for faster motions.

In Fig. 7, a linear relation between average pendulum length and apex height is observed. It shows that with longer pendulum, higher hopping height is obtained. It is also demonstrated that 1-step periodicity is obtained just by lower $\bar{l}_p$ and $\bar{y}_a$. This linear relation between pendulum length and resulted apex height is employed in Appendix A to show the stability of the system with the event based control approach for pendulum length adjustment.

The effect of pendulum length variations on system energy is illustrated in Fig. 8. The motions with higher energy are obtainable using shorter pendulums which produce faster motion with lower apex height, especially for 1-step periodic motions. Finally, the correlation between the horizontal velocity and the pendulum length is shown in Fig. 9, for periodicities larger than 1. It is observed that with the proposed technique no 2-step periodic motion is achieved. Between the remained ones the correlation of V_x and l_p for one period is very close to -1. It shows that the predicted relation between the pendulum length and horizontal speed is valid. Therefore, the pendulum length can be utilized to set the running velocity to a desired value.

IV. DISCUSSION

Most SLIP studies on running motion ignore the swing leg motion [8], [13]. When addressing swing leg motion, [16] applied subsequent modeling of stance and swing phase using one leg. This results in forward hopping instead of running. In this paper, a new approach is proposed for describing complete legs' movements during whole running stride.

The suggested controller is based on a pendulum like movement of the swing leg. Therefore, an oscillatory motion produced by a passive mechanism is the key idea of mimicking the leg behavior. In this regard, not only 1-step periodic motion was achieved, but also periodicity in more steps was obtained. This achievement is remarkable, while this method does not explicitly control the leg orientation or even use information about it.

In order to increase the stability of the motion in the manner of region of stability enlargement, an event based leg adjustment control approach was presented. It was shown that, although the initial running condition might not be kept, with an appropriate set of parameters, the stability is guaranteed. Hence, in the proposed control approach the only required feedback is the vertical velocity at takeoff which is

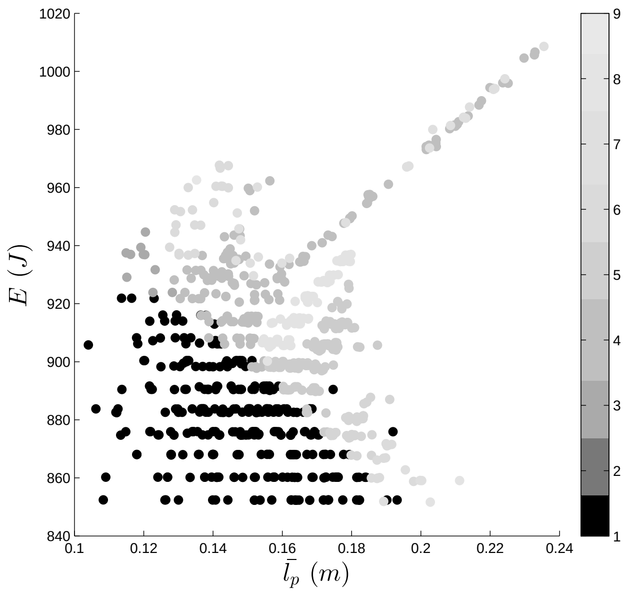

Fig. 8. Influence of pendulum length l_p on system energy E. Darker points mean smaller number of gait periodicity.

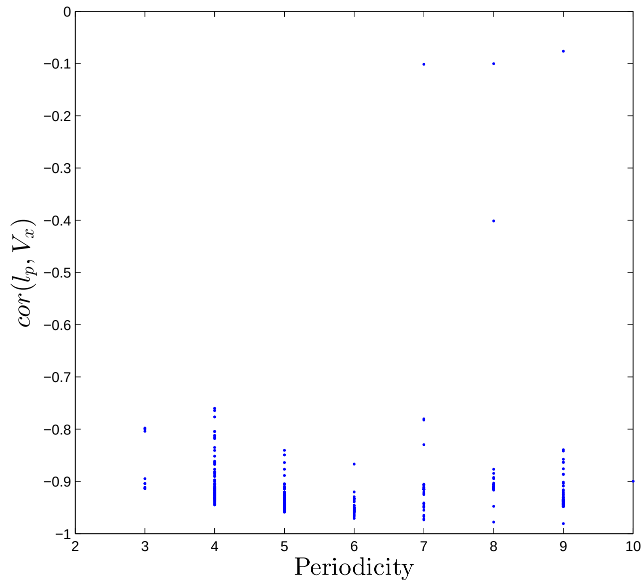

Fig. 9. Correlation between horizontal speed and pendulum length for different periodicities.

measured once at each step. Roughly speaking, a passive motion in low level is just tuned once in each step with a higher level feedback. This attitude is also close to human behavior when we never think during motion except for changing the speed or removing perturbations.

The results of our swing leg controller were also compared with human running behavior. Similar trends in leg angle patterns, especially some important features like leg retraction are of significant outcomes of this paper. The range for leg angle variation, the stance phase portion in whole stride and increment of leg movement with respect to horizontal speed increment are other similarities between human experiment and the simulated model. It can be concluded that even if the swing leg motion is not exactly pendulum like in human locomotion, it is produced by an oscillatory motion. On the other hand, if the pendulum motion is a proper model for swing leg movement, it might be possible to unify the complete leg motion with similar approaches like virtual pendulum concept which are already shown for stance leg [25]. Therefore, beside the virtual pivot point concept, presented in [25] (or divergent point in [26]) for balancing the upper body via mimicking the body motion during stance phase by a virtual pendulum, whole locomotion might be interpretable based on pendulum motion.

Appendix

A. Mathematical support of stability with Event based control pendulum length adjustment

From Fig. 7, a linear relation between the apex height and pendulum length is observed. Although this relation is not analytically provable because the SLIP model is not integrable [27][28], placement of the points close to a straight line is sufficient to consider the following equation.

$$y_{a(k+1)} = y_{ak} + \alpha \Delta l_{pk} \Rightarrow \dot{y}_{k+1}^2 = \dot{y}_k^2 + \alpha \Delta l_{pk} \tag{8}$$

In which $\dot{y}$ is the vertical speed at take off moment and index k and Δl_{pk} stand for the k^{th} step and the difference between pendulum length in two sequential stpes ($\Delta l_{pk} = l_{pk} - l_{p(k-1)}$). Considering Eq.(5), with some manipulations, this equation converts to

$$\dot{y}_{k+1}^2 = \dot{y}_k^2 + \frac{\alpha l_{p0}}{\sqrt{\dot{y}_0}}(\sqrt{\dot{y}_k} - \sqrt{\dot{y}_{k-1}}) \tag{9}$$

Substituting $\dot{y}_k^2$ in Eq.(9), by the same equation for in former step results in

$$\dot{y}_{k+1}^2 = \dot{y}_{k-1}^2 + \frac{\alpha l_{p0}}{\sqrt{\dot{y}_0}}(\sqrt{\dot{y}_k} - \sqrt{\dot{y}_{k-2}}) \tag{10}$$

Continuing this substitution until the first step, gives

$$\dot{y}_{k+1}^2 = (\dot{y}_1^2 + \frac{\alpha l_{p0}}{\sqrt{\dot{y}_0}}(\sqrt{\dot{y}_k} - \sqrt{\dot{y}_0}) \tag{11}$$

If there exists a limit cycle with a fixed pendulum length (l_{p0} and vertical speed $\dot{y}_0$, then we have $\dot{y}_0^2 = \alpha l_{p0}$. Therefore;

$$\dot{y}_{k+1}^2 = \overbrace{(\dot{y}_1^2 - \dot{y}_0^2)}^{\beta} + \sqrt{\dot{y}_0^3 \dot{y}_k} \tag{12}$$

It is possible to show that when k goes to infinity, the $\dot{y}_{k+1}$ converges to a fixed value (A) which is the root of the following equation:

$$P(A) = A^4 - 2A^2\beta - \dot{y}_0^3 A + \beta^2 \tag{13}$$

First suppose $\dot{y}_1 > \dot{y}_0$, which means β is positive. Computing polynomial P for $A_1 = \dot{y}_0$ and $A_2 = \dot{y}_0 + \sqrt{(\beta)}$ results in

$$P(A_1) = (\dot{y}_1^2 - \dot{y}_0^2)(\dot{y}_1^2 - 3\dot{y}_0^2) \tag{14}$$

$$P(A_2) = 3\sqrt{\beta}\dot{y}_0^3 + 4\beta\dot{y}_0^2 \tag{15}$$

If $\dot{y}_1 < \sqrt{3}\dot{y}_0$ then Eq.s.(14) and (15) have negative and positive signs respectively. Consequently, Eq. (13) has a root between A_1 and A_2. Similar argument is valid when β is negative considering $A_2 = \dot{y}_0 - \sqrt{|\beta|}$. In summary, defining error $\delta = sgn(\beta)\sqrt{|\beta|}$ (in which $sgn(.)$ is the sign function), the polynomial P in Eq.(13), has a root between $\dot{y}_0$ and $\dot{y}_0 + \delta$ if the vertical velocity at first takeoff is not too far from the desired value ($\dot{y}_1 < \sqrt{3}\dot{y}_0$). Therefore, the vertical speed converges to a value in the neighborhood of the desired value. This argumentation guarantees a considerably large region of attraction for the stable solution (limit cycle) of the controlled system.

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